GRAPHICAL METHOD

FOR THE ANALYSIS OF

BRIDGE TRUSSES;

EXTENDED TO

CONTINUOUS GIRDERS AND DRAW SPANS.

BY

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INTRODUCTION.

While bridge trusses for single spans have been treated of very fully by many authors, and the interrelation of their parts is well understood, those engineers who have had occasion to investigate the subject are fully aware of the difficulties which beset the attempt to determine the stresses on trusses continuous over the piers and on pivot or drawbridges. The mathematical investigations are intricate, and the formulæ deduced are troublesome in application. Most of the authorities pass over one or both of these problems with a judicious silence, and only by dint of hard study, and a diligent brushing up of mathematics in which the best of engineers are liable to become rusty, can any one follow the very few writers who do touch upon these subjects.

Prof. Clerk-Maxwell has given us a graphical method which, when applied to roofs and trusses under a steady load, leaves nothing to be desired. We also possess a method for graphically finding the stresses on single trusses, which, with some modifications, is explained and illustrated in the first part of this work; but its

extension and adaptation to continuous girders and draw-spans, as subsequently set forth, is believed to be entirely original.

The method will be found to be general in its application, easily put upon paper and well adapted for the drawing table and practical work. The instruments required are simple :—a good drawing-board, triangles, T-square or straight-edge, scale and lead-pencil.

In order to make all the steps of the process perfectly plain, especially to those unacquainted with graphical methods in general, and perhaps at the risk of dwelling too long on well understood points, we have commenced with the triangle of forces, whence we can probably all start together, and then have taken up shearing force and bending moment in a beam or truss ; trusses of single span with horizontal chords, with inclined chords ; trusses of two continuous spans, of several spans ; and finally pivot or draw spans.

If the reader will draw some of the following diagrams for himself, on a much larger scale, he will more readily see the truth of the statements which will be made ; and if we succeed in removing some of the mystery which surrounds these latter classes of bridges, our object will be accomplished.

CHAPTER I.

SINGLE SPAN TRUSSES: WITH HORIZONTAL CHORDS.

1. We know, by one of the fundamental theorems of mechanics, that, if three forces, not parallel, act upon a body, and are balanced, their directions must intersect at a common point, and that these forces must be proportional to the sides of a triangle drawn parallel to their directions; and that, conversely, if three forces be taken parallel to the three sides of any triangle, and proportional in magnitude to the same sides, with the direction of the several forces taken in the order obtained by passing over the sides of the triangle in succession, these three forces, applied at one point, must balance or be in equilibrium.

If the weight W, Fig. 1, be hung from two points, A and B, by the cords A C and C B, we may find the pull or tension on each of the cords, by drawing a vertical line *a b*, equal, by any convenient scale, to the given weight W, and then drawing the lines *a c* and *b c*, from the extremities of *a b*, parallel to A C and B C, intersecting at *c*. Then will *a c* and *b c* represent, by the same scale by which W was laid off, the pull on the cords A C and B C. The arrows represent the directions of the stresses relatively to the point C, and the arrows on the triangle *a b c* must

be found to point successively in order round the triangle.

If W and the two stresses *a c* and *b c* had been given, we might have reversed the problem, and found the direction and length of two cords which, while supporting W, would have exerted the given tensions on the points A and B. As it is possible to draw any number of triangles on the line *a b*, we may have a like number of arrangements of cords from A and B to carry W.

2. If we have several weights suspended from the points A and B, Fig. 2, by means of a cord whose weight may be neglected or considered to be included in the given weights, we find the tensions on the several portions of the cord by the successive applications of the above process. The weight at C is balanced by the tensions of A C and C D, and we may draw the triangle of forces for the point C. Next, having the weight W_2 and the tension, just found, on C D, we may draw a triangle for the point D.

But we may bring the triangles together, into one figure, by the following construction. Draw a vertical line 1–2, and measure on the same W_1, W_2, W_3 and W_4, successively, to any convenient scale of pounds or tons to the inch. Draw 1–0 parallel to A C and 2–0 parallel to B F. Connect the point 0 with the other points of division on the vertical line. Then will *a–c*, *c–d*, *d–e*, *e–f* and *f–b* be equal to the stresses on A C, C D, D E, E F and F B, and the portions of the cord must be

parallel to the lines radiating from 0, or the cord will not otherwise be in equilibrium.

The figure 0 12 will be called the *stress diagram*, the line 1–2 the *load line*, and the cord from A to B, which hangs in what is called a *funicular polygon*, we shall have occasion hereafter to designate the *moment curve*.

If the weights and their horizontal distances from the point A and the horizontal distance of B from A were alone given, we might draw a *moment curve* to satisfy the condition of equilibrium, by assuming the point 0 in any convenient position, drawing the radiating lines to the several points on 1–2, then drawing a line parallel to *a–c* from A to meet a vertical through W_1, from the point C, so found, drawing a line parallel to *c–d* to meet a vertical through W_2, and so on, the last line, parallel to *f–b*, determining the position of B, since its horizontal distance from A was supposed given. This latter simple application of the method for finding a moment curve will suffice for the investigation which is to follow.

3. If all the forces act on a structure in one vertical plane, they may be resolved into horizontal and vertical components in that plane. The external forces considered here are all vertical, being the weights of the structure and the load, together with the reactions of the points of support which balance them. If a beam or truss, supported in any manner, is cut by a vertical plane, we shall have, on one side of the plane of section, the resultant of all the external vertical forces which act upon the portion

of the beam upon that side, and this resultant must be balanced by the sum of the vertical components of the stresses in that section of the beam, or by what is commonly called the *shearing stress;* and the resultant of vertical forces just spoken of is also called the *shearing* force at the section in question. Further, the external forces on one side of the plane of section, (since they are vertical, they have no horizontal components;) have *moments* around the plane of section, and the resultant moment, called the *bending moment*, is obtained by multiplying each force by its distance from the plane of section as a lever arm, adding together those products which have a tendency to rotate in one direction, and subtracting the sum of those products which tend to rotate in a contrary direction. The remainder is the bending moment at the section and tends to produce rotation in the direction shown by the larger sum. This moment is balanced by the *moment of resistance* of a couple made up of the respective resistances to compression and extension of the fibres or pieces in the two portions, upper and lower, of the beam or truss, multiplied by the distances between the sets of particles undergoing equal opposite stresses. Knowing the shearing force and bending moment at different sections of a beam or truss, we must have forces exerted, through the portion cut by the plane of section, sufficient to balance them.

4. Suppose now a beam, such as is represented in Fig. 3, to be supported at the two ends, and to have four un-

equal weights situated, as shown, at unequal intervals upon it. Let the weight of the beam be included in the imposed weights. It is required to find the *supporting forces*, or the pressures P_1 and P_2, upon the abutments A and B, the shearing force and the bending moment at every point.

Draw a stress diagram, as before, by laying off, on a vertical line, 1–2, Fig. 3, the weights or loads W_1, W_2, etc.; assume a point 0 at a convenient distance from 1–2, and draw radiating lines from 0 to all the points of division on 1–2. Draw vertical lines through the points of support and the loaded points, A, C, D, E, F and B. Commence at some point, A′, in the vertical through A, and draw A′ C′ parallel to 0–1. From the point C′, where this line cuts the vertical through the first weight, draw C′ D′ parallel to 0–3. Continue the same process until the last line F′ B′, drawn parallel to 0–2, meets the vertical line let fall from B, the second point of support.

If A′ C′ D′ E′ F′ B′ were a cord, fastened at A′ and B′, and under the action of the given weights in their given positions, it would be in equilibrium, as shown in a preceding section. The pull on A′ would be in the direction A′ C′ and of the amount 0–1. The pull on B′ would be in the direction B′ F′ and of the amount 0–2. If now the two ends of the cord, in place of being fastened at A′ and B′, were attached to the ends of a rigid bar, A′ B′, the whole system might be suspended by two cords from the points A and B without disturbing the equilibrium of the

polygon. For, if we draw 0–5, in the stress diagram, parallel to $A' B'$, we can see that the inclined pulls at A' and B' are, by the introduction of the bar $A' B'$, each decomposed into a thrust along the bar and a force acting vertically downwards, as shown by the arrows. At the point A' we have three forces which, in the stress diagram, must make the triangle whose sides are 0–1, 0–5 and 1–5. In the same way, considering the point B', we shall have, in the stress diagram, the triangle whose sides are 0–2, 0–5 and 2–5. The thrust 5–0 from A' balances the thrust 0–5 from B', and there remain 1–5 and 5–2, or P_1 and P_2, the forces exerted by this system on the points of support A and B. Any other system of framing, loaded in the same way, must give the same pressure on the points of support, and the reactions of the abutments will be equal and opposite to these pressures.

Therefore, to find the supporting forces :—having drawn the moment curve, connect A' with B', draw 0–5 parallel through 0, and the two portions into which the load line is thus divided will be the forces required.

5. As the shearing force at any section is the resultant of the vertical forces on one side of the plane of section, the shearing force at any point between A and C will be P_1, the only vertical force acting on the left of that point. As P_1, the reaction of the abutment, acts in an upward direction, draw $A g$ vertically, equal to P_1. Between C and D the shearing force will be $P_1 - W_1$, or 3–5 ; between D and E it will be $P_1 - W_1 - W_2$, or 4–5 ; and so

for all parts of the beam. The shearing forces can be read off directly from the load line, as just shown, the point 5 being always one end.

But, if we draw from g a line $g\,i$, parallel to A B, then at the point i, vertically over C, measure off, downwards, $i\,k$, equal to W_1, draw $k\,l$ horizontally, then $l\,m$, equal to W_2, and then $m\,n\,p\,q\,r\,s$, we shall have a broken line, the ordinate to which, from any point of the beam A B, will be the shearing force at that point. When the W's which have been subtracted exceed P_1, the broken line passes below A B, and finally, on arriving at B, having subtracted all the W's, we have a shearing force equal but opposite to P_2. So, at any section, the shearing force at one side of the plane of section, obtained by the subtraction just described, will always be equal and opposite to the shearing force on the other side of the same plane of section, obtained by working from B, with the supporting force P_2 as the minuend. Such a result is required, to fulfil the condition of equilibrium.

6. Lastly, to find graphically the bending moment at any point. Take the point S. The bending moment on the beam at S is

$$P_1 \,.\, A\,S - W_1 \,.\, C\,S - W_2 \,.\, D'S.$$

Drop a perpendicular from S, cutting the polygon A′ E′ B′ at I and K. Produce A′ C′ and C′ D′ to meet this perpendicular at L and M. Also draw C′ N horizontally. In the stress diagram draw a horizontal line 0–6,

through 0, to meet the load line. Call this line H. It is the horizontal projection of the stress on each side of the funicular polygon, or moment curve, and is plainly equal to the horizontal stress at any point of the curve, which is well known to be *constant* for a system of vertical loads.

The triangles C′ N M and 061, having their sides parallel, are similar, and we have the proportion

$$M\,N : C'\,N = 1\text{–}6 : 0\text{–}6.$$

From the similar triangles C′ L N and 036 we have

$$L\,N : C'\,N = 6\text{–}3 : 0\text{–}6.$$

Hence

$$\frac{M\,N - L\,N}{C'\,N} = \frac{(1\text{–}6)-(6\text{–}3)}{0\text{–}6},$$

or

$$M\,L\,.\,(0\text{–}6) = (1\text{–}3).\,C'\,N.$$

But

$$0\text{–}6 = H\,;\quad 1\text{–}3 = W_1,\ \therefore$$
$$H\,.\,M\,L = W_1\,.\,C'\,N = W_1\,.\,C\,S.$$

In the same way

$$H\,.\,L\,K = W_2\,.\,D\,S, \text{ and } H\,.\,I\,M = P_1\,.\,A\,S\,;\ \therefore$$

the bending moment

$$M = H\,(I\,M - M\,L - L\,K) = H\,.\,I\,K.$$

Hence the bending moment, at any point of the beam, is proportional to the ordinate, from A′ B′ to the moment curve, vertically below that point, and is equal to the product of that ordinate by H, the constant horizontal component of the tensions on the curve.

The ordinate must be measured by the scale to which the beam is drawn, and the line which represents H by the scale to which the load line is measured off; but the two figures may be of the same scale, that is number of tons and feet to the inch, or of different scales, whichever is most convenient. If the point 0 be taken at such a distance from the load line that 0–6 shall measure just ten or one hundred units of weight in length, the bending moment will be readily obtained by scaling the ordinate, and moving the decimal point to the right, one or two places as the case may be.

It will be noticed, that the greatest value of the shearing force occurs where the bending moment is least, and *vice versa.*

7. We may now proceed to apply this method to a bridge truss, with parallel chords, under a given moving load. Let the truss be represented by A B C D, Fig. 4, supported at A and D. The span is supposed to be 80 feet, height of truss 10 feet, fixed load $\frac{1}{4}$ ton per running foot, moving load $\frac{1}{2}$ ton per running foot. The scales of the figure are, as shown below it, 30 feet = 1 inch, and 30 tons = 1 inch. The small weights represent the fixed load, arising from the truss and platform, as if concentrated on the joints of the lower chord; the larger weights represent the moving load. As each panel is ten feet long, the load from the truss at each joint will be $2\frac{1}{2}$ tons, and from the rolling load 5 tons. The points A and D will each carry one half a panel weight; these weights will cause no stress in the truss, and might be neglected

altogether; but it will be found convenient to plot them on the load line, as thus the total weight of the truss will be accounted for, and the shearing force will be more readily obtained.

Suppose the rolling load to extend from the abutment A to the joint J, inclusive. Draw a vertical line, and lay off 1–2 equal to $\frac{2\frac{1}{2}+5}{2}$ tons, the weight at A; next 2–3 $= 7\frac{1}{2}$ tons at E, 3–4 $= 7\frac{1}{2}$ tons at F, and so on to 6–7 at J; then 7–11 $= 2\frac{1}{2}$ tons at K, 11–12 $= 2\frac{1}{2}$ tons at L, and finally $1\frac{1}{4}$ tons at D, reaching a point 13, midway between 12 and 14. Next assume the point 0, which is here taken so that H measures 20 tons. Leave out of consideration 1–2, the weight at A, and starting at A′, a convenient distance vertically below A, draw A′ E′, E′ F′, ... L_1' D_1', parallel respectively to 2–0, 3–0, 4–0, &c., the line L_1' D_1' being drawn parallel to 12–0.

Connect A′ D_1', and draw, in the stress diagram, 0–9 parallel to A′D_1'; 1–9 and 9–13 will be the supporting forces at A and D. Lay off vertically, from a line *a d*, *d g* equal to 9–13, and *a n* equal to 1–9. The shearing force at any section might then be represented by a broken line, as in Fig. 3, *g i k l m*, &c.; but we can more readily draw the line *g p n*, which would be the bounding line of all the shearing ordinates, in case the load were uniformly distributed, instead of being concentrated at a series of points. If we wished to draw the broken line, we should deduct from *d g* the load on D, and then draw a horizontal line across the space under the first panel, which line would cut *g p* at a point vertically above *q*, or in the

middle of a panel space. That this ought to occur may be seen, if we consider that the load on each joint comes from half of the panel on each side. The inclination of $g\,p$ must be such, that the ordinates to it shall diminish $2\frac{1}{2}$ tons for each panel; the ordinates of $p\,n$ must also diminish at the rate of $7\frac{1}{2}$ tons per panel; and, if $g\,p$ and $p\,n$ are drawn from g and n respectively, then p will come in the middle of a panel.

8. If the rolling load retires, so that G is the last fully loaded point, we shall have three loads of $7\frac{1}{2}$ tons each, and the points from I to L will carry only $2\frac{1}{2}$ tons each. We may make available a large portion of the curve already drawn; since 4–5 represents the load on G, we can lay off the smaller loads below, the one on L falling at 6–16. Constructing a second moment curve, as we did the first, we shall get A' E' F' G' I' J_2' K_2' L_2' D_2', and, drawing 0–8 parallel to A' D_2', we get the new supporting forces for this position of the rolling load. Hence we find $h\,r$ and $r\,i$ as we did $g\,p$ and $p\,n$.

If the load extends over the whole truss, our load line will be 1–10, and the moment curve A' E' F' G' I' J' K' L' D'. As 0 was taken opposite the middle of 1–10, and as, for an entire uniform load, the two supporting forces are equal, the line A' D' is horizontal. The shear diagram for this case is $d\,c\,v\,b\,a$.

9. If we should carry out the construction for every position of the load, we should discover some facts which are of importance as rendering needless the drawing of

so many diagrams; and we may see indications which lead to that discovery in the figure already described. First:—the bending moment, at any point of a beam or truss, is greatest when the whole beam is loaded. The curve A′ G′ J′ D′ will give ordinates of the greatest length, and these ordinates, multiplied by H, give the bending moments. Second:—the shearing force at any point is greatest when the longer segment, of the two into which the point divides the truss, is fully loaded. For example, the shear in the panel J K is greatest when the rolling load extends from A to J, at which time the ordinate will rise to *p*; but, for any other position of the load, the ordinate will be less.

Third:—we should find, in addition, that the points *p*, *r*, &c., lie on a parabola whose tangents are readily drawn; and the curve shown by the dotted line from *m* to *t*, can then be constructed.

The limiting values of the shearing force at the abutments are, for the least value, *one half of the weight of the truss unloaded*, and, for the greatest, *one half of the weight of the truss and full load.* Lay off *a e* equal to the former amount and *d c* equal to the latter If *d c* be plotted above *a d*, *a e* must be plotted below; connect *e* and *c* with the middle point *v* of *a d*. Otherwise, the shearing diagram for the loaded truss will be *d c b a*, and for the unloaded truss *d f e a*; all other diagrams must have their terminal points between *f* and *c*. and *e* and *b*; we have, then, the tangents *c v* and *e v* on which to construct the parabola.

10. The curve is easily found as follows:—let A B and B C, Fig. 5, be two tangents. Divide each tangent into the same number of equal parts; number the points of division 1, 2, 3, &c., as shown. Draw straight lines 1–1, 2–2, 3–3, &c., and the curve sketched tangent to these lines will be the parabola required. In Fig. 4, each tangent is already divided by the verticals into four equal parts; if each part be now bisected, a sufficient number of points will be obtained, and the lines then drawn will limit the ordinates without sketching in the curve. The ordinates in the end panels will thus terminate at *m* and *t*, the middle points of the panels, (as required by § 7, end).

11. The shearing force in any panel is then obtained, for example in the panel K L, by erecting the ordinate *l s* to the curve, at the middle point *l* of the panel length. The stress on the tie S K is then easily obtained by drawing *s k*, parallel to S K, from *s* until it meets the horizontal line at *k*. The stress on the diagonal will be *s k*, and on the post S L it will be *s l*, to the same scale as *c d* or *a e*, and so for the other panels The point *r* under the middle of the panel G I, will be the last one where we shall find any shearing force in an upward direction on the right hand side of the plane of section, when the load extends from A to G and in the panel G I is therefore found the last necessary tie parallel to C L.* The remain

*Many engineers carry the diagonals sloping one way through all of the panels, for practical reasons. But they are not essential beyond the point shown, and, as they do no real work, may be made as small as is thought desirable.

ing panels from G to K would require diagonals, if ties, sloping in the contrary direction, and as, when the load goes off in the other direction, we shall get similar but greater stresses in these panels, exactly such as we have found from D to G, it is sufficient to find the stresses from D to G, and then make the truss from A to J symmetrical with D G. The heavier the rolling load compared with the fixed load, the further will the diagonals sloping one way pass beyond the centre.

If a diagonal slants up towards the abutment to which its stress goes, it is a tie; if it slants down, it is a strut. If the load were placed on the top chord, the ordinate $z\,l$ would be the stress on R K. In short, from any ordinate to the parabola at the middle of a panel, we determine the stress on that diagonal and vertical which, taken together, may be considered to connect two adjacent weights.

12. It is also unnecessary to draw any moment curve besides A′ F′ I′ K′ D′. The bending moment on the truss is taken at each panel joint. If we pass a plane of section, for example one through the joint K, we shall have, taking moments on the right of and about K, the thrust on the top chord in the panel R S, multiplied by the height of the truss R K, as the moment of resistance to balance the bending moment produced by the external forces. As the vertical R K and the diagonal S K both terminate at the proposed moment point or axis K, the stresses along those pieces cause no moments. Similarly,

taking moments round R, we have the tension on J K, multiplied by R K, to balance the bending moment. Therefore the compression on R S equals the tension on J K, and the same equality is true of S C and K L, and of all the other portions of the chords. Of course the compression or tension is constant for a panel length. There is no stress on L D or A E, as these pieces are unnecessary for equilibrium. The compression on the two panels of the top chord at the middle does not call for any equal tension on a panel length in the bottom chord, for the two ties which meet at I act directly against each other.

Now, as the maxima stresses on the chords occur when the entire span is loaded, to find the tension and compression on the bottom and top chords between any two adjacent diagonals which incline the same way. it is only necessary to take the ordinate to the parabola under the common panel joint of the two diagonals, multiply it by H from the stress diagram and divide by the height of the truss ; the result will be the required tension and compression.

13. As we may draw the shearing diagram required for the solution of this case without employing a stress diagram, we may also draw the moment curve alone ; or better, if the truss has parallel chords, as now supposed, we may draw a curve whose ordinates shall give directly the required tensions and compressions. For, H and the height of the truss being constant, these forces are direct-

ly proportional to the bending moment M. Call the span l, and the height of the truss k. It is well known that the bending moment at the centre of a beam supported at both ends, and uniformly loaded with a total weight (of truss and load), W, is

$$M = \frac{W l}{8}.$$

Therefore the stress required at the centre will be $\frac{W l}{8 k}$. As the bending moment varies as the ordinates to a parabola, so will the value for the chord stress vary. Therefore compute the ordinate for the middle point of the truss

$$\frac{W l}{8 k},$$

and, through the point thus found and the two ends of the diameter of length l, draw a parabola. To readily construct the required curve, (see Fig. 5,) taking D E equal to l, at the middle point G lay off G F equal to $\frac{W l}{8 k}$. Draw verticals through the panel joints, D H I K. Lay off D S = G F, and divide it into the same number of equal parts with D G. Draw F O, L P, M R and N D. Complete the other half of the figure, if thought desirable, in the same way. The points F, L, M, N and D will lie on the curve, and the stresses on the chords will be, in succession from the abutments to the middle, H N, I M, K L and G F.

Each part of the truss which undergoes tensile stress should have its effective section in square inches equal to

the maximum stress it must exert divided by the safe working stress on the square inch. The cross-section of pieces in compression should be determined by Gordon's Formula or some similar method. The working out of details does not come within the scope of these pages.

CHAPTER II.

SINGLE SPAN TRUSSES:—WITH INCLINED CHORDS.

14. While the determination of the stresses in a truss with chords which are inclined or curved is not quite so simple, it involves no particular difficulty, the chords being considered as jointed at each intersection. Take, for example, the truss represented by A B C D, Fig. 6. The span and the load are taken the same as those for the truss, Fig. 4, just described. The bending moment is obtained as before for the truss loaded throughout its length, and the stress diagrams of Fig. 4 apply to this case also. The bending moment at any panel joint, being obtained from the proper ordinate of the curve A′ F′ I′ K′ D′ multiplied by H, must be divided by the height of the truss at the panel joint in question, and we shall then have the *horizontal component* of the pull or thrust on the chord. Multiply this horizontal component by the length of the inclined chord in the panel and divide by the horizontal distance between the panel joints; or, if we call the length, horizontally, of a panel a, and the difference of level of the two joints in the same chord b, multiply the horizontal component above by

$$\sqrt{1+\frac{b^2}{a^2}},$$

and we shall have the stress on that piece of the chord.

15. To find the stress on any diagonal, such as O P, with the load as sketched in the figure, covering O for the last point, we must draw the moment curve $A' F' I' K' D_1'$, Fig. 4. Then consider that a plane of section has been passed through the panel O N P Q, Fig. 6. An enlarged view of this panel is drawn just below on the left. On one side of this plane, say the right hand, we have the vertical shearing force, the horizontal pull through Q and the horizontal compression which would act through N. On the other side of the plane we find three inclined pieces whose stresses must balance the rectangular components on the first side.

Now, from the moment curve for the given position of load, Fig. 4, take N J′, multiply by H, divide by O N, and lay off the quotient, which is the *horizontal* stress through N, horizontally at *v y*.* In the same way P K′ . H, divided by P Q, gives us the horizontal stress through Q, laid off at *v x*. The vertical shearing force will be the ordinate to *p*, Fig. 4, and is plotted vertically upwards at *v u*. Now draw *n p* from *u*, parallel to N P, and limit it by a vertical from *y*; draw *o q* from *v*, parallel to O Q, till it meets the vertical through *x*; draw the line *o p*, and it must be parallel to O P, or some error in construction has been made. Thus we have a check on the accuracy of our work. By dispensing with this check it is only necessary to determine one of the horizontal stresses, and

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* Diagrams I. to IV. of Fig. 6, are enlarged one and one-half times as compared with the lines obtained from Fig. 4, as shown by the attached scales.

then we may draw the other lines parallel to the respective pieces. Indeed it would be possible, by determining both of the horizontal stresses, to dispense with the shearing diagram altogether; but it would not be advisable so to do. The required stress on O P is *o p*. The stresses on *n p*, *p o* and *o q* must follow one another round the figure, as shown by the arrows, which represent the actions of the pieces against the plane of section, as shown in the enlarged sketch of this panel, and it will be noticed that the horizontal projections of *n p*, *p o* and *o q* balance; while their vertical projections are in equilibrium with the shear *v u*.

16. It remains to find the stress on P Q. Consider the joint P. This joint is in equilibrium under the action of four forces exerted by the four pieces which meet at that point. These forces must form a closed polygon, and, as we have already determined two sides, the remaining ones are readily drawn. The stresses on N P and O P being represented by *n p* and *o p*, draw *p q* parallel to P Q and *p r* parallel to P R. The required stress on P Q is *p q*. The arrows in this quadrilateral show the direction of the stresses exerted on P by the several pieces, and they must proceed in order round the figure. The horizontal projection of the stress on PR is the same as that of the stress on O Q, which result would be expected. The stresses *n p*, *p r* and *o q* are the ones existing in the pieces to which they refer for the present load; but, as they are not the greatest stresses which the chord pieces

undergo, they are of no special value here. The maxima stresses were determined in § 14.

17. The portion of the figure within the triangle $t\,d\,y$ gives a graphical method for finding the horizontal components $v\,x$ and $v\,y$ without calculation. For example: since

$$v\,y = \frac{\mathrm{N\,J'} \, . \, \mathrm{H}}{\mathrm{O\,N}},$$

draw $t\,d$ at any convenient angle with the horizontal line, make $t\,v = \mathrm{H}$; $t\,b = \mathrm{O\,N}$; $b\,d = \mathrm{N\,J'}$ (Fig. 4); draw $b\,v$, and, parallel to $b\,v$, through d draw $d\,y$; $v\,y$ will be the required component. For, from similar triangles,

$$t\,b : \mathrm{H} = b\,d : v\,y = \frac{b\,d \, . \, \mathrm{H}}{t\,b}.$$

Similarly, $t\,a = \mathrm{P\,Q}$; $a\,c = \mathrm{P\,K'}$ (Fig. 4); and $c\,x$, drawn parallel to $a\,v$, determines $v\,x$. If $c\,x$ and $d\,y$ cut the horizontal line at favorable angles, the values of $v\,x$ and $v\,y$ may be thus obtained quite satisfactorily.

18. If the moving load only extends to and includes J K, we construct, in the same way, diagram II., using the ordinates R G′ and M′ I′ from Fig. 4. In this case $k\,m$ runs below the horizontal line, but the remainder of the figure is similar to the one just described. It will be noticed that the inclinations of the chord pieces J L and K M now act to increase the vertical force transmitted *towards* the abutment D. The stress on any diagonal is *increased* by such an inclination of either chord as tends to make the *height* of the panel, on the *side* of the abutment

toward which the diagonal conveys its load, *greater;* and *vice versa.*

After we pass the point where the parabola for shear, Fig. 4, passes below the horizontal line, the vertical for shearing force must be drawn *down* from the horizontal line in Fig. 6. Then, taking the panel G J K I, and using the dotted curve of Fig. 4, we draw $g j$, $i k$, and $i j$, diagram III., exactly as before. Since $g j$ and $i k$ cross, we still have a pull on the diagonal I J, as shown by the arrows; and we may also find a compression on J K, in case the inclination of J L does not take all of the remaining vertical force. An inspection of diagram III. will render this statement clear. If we advance one panel nearer abutment A, the moving load now being considered to rest on A and F only, the panel F E G I will give us diagram IV. On attempting to find the stress on a diagonal from F to G, we shall see, by the necessary direction of the arrow on $f g$, that a piece F G would thrust against the plane of section. As our diagonals are, in this example, supposed to be ties, we have passed the limit where those sloping in this direction are required. A similar set, from A to Q, will complete the truss. If the truss is deeper at the centre than at the ends, the effect of the inclination of the chords is to require more diagonals sloping one way than a truss with parallel chords will need. A bowstring girder, jointed at panel points, requires diagonals throughout.

19. The stress on any diagonal, such as O P, will be

found to be a maximum when the load extends from the abutment A to the panel joint at its foot, as has been previously stated to be the case with a truss having parallel chords. If the diagonals had been struts instead of ties, no difficulty would have been experienced ; the line *o p* which now slants one way would in that case have slanted in the opposite direction. If the load were on the top chord, instead of the bottom as here, in place of finding the stresses on O P and P Q, we should find them on O P and O N ; that is :—take the vertical which meets the unloaded end of the diagonal in action. The change in diagram I. would be that, instead of drawing *p r* and *p q*, we should prolong *o q* (shown by a dotted line,) to meet the vertical through *y*, and the upper intercepted portion of this vertical would be the stress on O N.

By the method here given, of drawing the horizontal stresses on the right of the vertical line for shearing force, when the latter acts upward we have the stresses on the top and bottom chords in the relative positions of the pieces themselves, that is *n p* is over *o q*. The bending moment tends to produce convexity downwards. We shall, in the latter part of the book, have occasion to refer again to these diagrams ; the bending moment in that case being in the contrary direction, we shall then plot the horizontal stresses to the left of the vertical *u v*.

CHAPTER III.

CONTINUOUS GIRDER OF TWO SPANS.

20. Let us pass next to the case of a beam supported at two points, loaded at intervals, and overhanging one of the points of support. Let A I, Fig. 7, be the beam, supported at A and B, and let the weight of the beam be considered as concentrated with the additional loads. Draw the stress diagram 012, as in other cases. Commence at A′, and draw A′ C′ D′ E′ F′ G′ I′ parallel to the radiating lines of the stress diagram, the angles occurring on the verticals let fall from the *weights*. There will be one line, parallel to 0–2, still to be added, and this line should be drawn from I′ to the vertical through B, as in former cases, but to B′, in the reverse direction. (Compare Fig. 3.) Connect A′ B′ and the moment diagram is complete. A line through 0, parallel to A′ B′, will divide the load line into the two supporting forces, P_1 at A and P_2 at B.

If we lay off P_1 at A, equal to A *a*, and then construct *a c d e*, etc., as was done in Fig. 3, we shall reach a point *b* under B; now, commencing at I, lay off I *i* equal to the weight at I, then add the weight at G, finally reaching the point *b′*. B *b* + B *b′* must be equal and opposite to P_2, which acts upward. Drawing the horizontal line marked H through 0, we find the bending moment at any point by multiplying H by the ordinate between the moment curve

and the line $A' B'$ or $B' I'$ for that point. At K', there being no ordinate, the product is zero ; consequently the beam is not bent at K. As we pass from K' to B' and I', the ordinate, being below the curve, may be called negative ; the bending moment is in the contrary direction over the portion K I, from that existing over A K, and it tends to produce convexity on the upper side of the beam, reversing the tension and compression on the fibres. The point K is called a *point of contra-flexure.* The curvature of the beam is shown, to an exaggerated scale, by the dotted line A L B M.

In case the beam overhangs both points of support we may have two points of contra-flexure ; but the overhanging portions may have sufficient weight to cause convexity upwards over all the intermediate portion, when the line corresponding to $A' B'$ will pass entirely below the curve and there will be no points of contra-flexure. If the two points of support are brought together into one point, and the two overhanging portions balance each other, we have the case of a pivot or swing bridge when open. This case will be further considered in the sequel.

21. If we next turn our attention to a girder continuous over two spans, the simplest case of which is a beam resting on three supports, we shall find that a partial solution of the problem by mathematical calculation is attended with considerable difficulty, and that a complete solution for the bending moment and shearing force at every section, under moving, partial and irregular loads, taxes the

powers of the best mathematicians and is well nigh impossible, on account of the complexity of the formulæ, so far as any practical application of them by the engineer is concerned. Where each span has a uniform load over its whole extent, although different spans may have loads of the same or different intensities, Clapeyron's "Formula of the Three Moments" will readily give us the moments over the piers for a continuous girder, and hence we can obtain the moments at other points; and, if the beams are symmetrically loaded, or so loaded as to be horizontal over the piers, the investigations are not difficult and can be found in most text-books treating of the flexure of beams. But no book in common use gives us any method for determining the shearing stress under a partial load, a determination which is necessary before the bracing can be correctly proportioned.

We propose now to develop a graphical method which will be readily applicable to all cases of continuous girders, under discontinuous and moving loads, and by which we can as easily determine the maxima stresses on the chords and bracing of any truss, loaded in any manner, as we have done for single spans in the preceding pages. The diagrams will show, what might have been expected from analogy, that the maxima stresses on the chords of continuous girders occur when one or more spans are fully loaded, and the maximum stress on a brace occurs when the moving load covers one of the segments into which the piece under consideration divides the span; but, in both cases, the stresses in one span are very materially altered

by the presence or absence of a load on the adjacent spans.

22. Suppose that we have a beam, as represented in Fig. 10, supported on two abutments, A and C, and divided by the pier at B into two unequal spans. Its own weight is uniformly distributed, and it carries, in addition, a uniformly distributed load of twice the intensity from C to D and also from E to F. Divide the two spans into convenient parts, equal or unequal (here taken equal), and consider the load to be concentrated at the points of division. Draw verticals through these points, and, having constructed the load line and stress diagram 012, draw the moment curve between C′ and A′, as in previous examples. The loads at A and C are neglected, and are represented by the portions of the load line which project at 1 and 2 ; since B carries a load, the vertical through B will determine one of the angles of the moment line, in the same way as any other loaded point.

To complete the construction we must draw A′ B′ and C′ B′. We know that B′ must lie below the curve (for we have a negative moment of flexure over B), and we usually have two points of contra-flexure, where A′ B′ and C′ B′ cut the curve. We must have, further, some condition to limit the position of B′, since there is manifestly but one correct value for the moment of flexure over the pier for a given position of the load. As no bending moment exists at A, A′ B′ must start from A′ ; and, similarly, C′ B′ is drawn through C′.

23. From the bending moment there arises in the beam a change of inclination at successive points, and a deflection, either *positive* or *negative*, of all portions of the beam from the original horizontal line. For the bending moment at any point, being opposed by the moment of resistance of the beam, causes an elongation and compression of the fibres on opposite sides of the beam, proportional to the bending moment. This elongation and compression causes (see Fig. 8) a change of direction of the centre line of the beam, in a vertical plane, at that point. Therefore the change of direction or inclination at any point is proportional to the bending moment at that point, or to the ordinate between the moment curve and the straight line, (as A′ B′, Fig. 10), and the total change of inclination between any two points is proportional to the sum of all the bending moments between those points, or to the area included between the *moment curve*, the straight line from the abutment to the pier and the two limiting ordinates under the points.

By reference to Fig. 8, we can see the change of inclination produced in the beam at A, by the elongation of the upper fibres at that point and the compression of the lower ones. We can also see the effect of successive changes of inclination, at B, C and D, in altering the direction of the beam, and we must note that the changes of inclination at E, F, &c., produced by bending moments in the opposite direction, tend to bring the beam back towards its original *direction*. Consequently, the change of inclination between any two points is proportional to the

algebraic sum of all the ordinates to the moment curve between those points; and, in Fig. 10, supposing C′ B′ and B′ A′ to be correctly placed, if we cut off an area from G′ towards B′, equal to the area between A′ and G′, the centre line of the beam, at the point vertically over the dividing ordinate just drawn, will have its tangent parallel to the tangent at A. If we know any point where the beam is horizontal, we can thus find the other points where it is horizontal; but the determination of this question is not material to our problem.

24. The *deflection* of any point of a beam from the original line depends upon the changes of inclination and the distances of the points at which they occur from the first point. Thus, if the originally straight stick $a\,g$, Fig. 9, is bent at a, the point g will be carried to a point on the line $a\,l$, the distance through which it is displaced depending upon the angle at a and the distance $a\,g$; if another angle is made at b, the point g will now be found on $b\,i$, and, on a further bending at c, it will move to the direction $c\,h$. The changes of inclination at d, e and f, in the contrary direction, will carry the point which was originally at g, through $d\,k$, $e\,m$, $f\,g$, finally back again to g. (The deflections of all beams and trusses are so small that the curved line of a beam under a load is always considered practically equal in length to the horizontal distance between the two points of support.)

We see, then, that the position of a point D, Fig. 10, in a beam under flexure, in reference to some point, such as

C, as origin, and from a tangent to the beam through that origin, depends upon the successive changes of inclination between the two points and the distances from the point D at which they occur, regard being paid to the direction of the change of inclination. Then, as each change of inclination is proportional to the ordinate to the moment curve, the deflection of a point from a tangent through the origin is proportional to the sum of the products of each ordinate into its distance from the point in question ; or, as the horizontal distances are the proportional projections of the distances on the tangent, and as the sum of each ordinate multiplied by its horizontal distance from D is the same thing as the area between C′ and D′ multiplied by the distance of its centre of gravity horizontally from D′, *the deflection of D from the tangent through C is proportional to the area C′ D′ multiplied by the horizontal distance of its centre of gravity from the vertical through D.* Then, if C L is the tangent through C, the deflections of the points B and A, with reference to C, will be proportional to B K and A L, or, from the similarity of triangles, to B C and A C, two known quantities.

25. Therefore the two lines A′ B′ and C′ B′ must be drawn to a point B′, so situated that (denoting the centres of gravity of the respective areas by *a*, *b*, *c* and *d*), the areas

$$\frac{\mathrm{C'P'D'} \, . \, de - \mathrm{K'D'B'} \, . \, ci}{\mathrm{C'P'D'} \, . \, df - \mathrm{K'D'B'} \, . \, ch - \mathrm{K'B'G'} \, . \, bh + \mathrm{A'F'G'} \, . \, ag} = \frac{\mathrm{BC}}{\mathrm{AC}}.$$

All of these quantities are readily measured and com-

puted, as will soon be shown; therefore we may draw A′ B′ and C′ B′ as trial lines, as near the right position as we can, and then compute the first ratio. If it does not equal the second ratio, move B′ and try again; a second approximation will generally be sufficient. Call any area, as above, multiplied by its distance from a certain point, an *area moment.*

We may, with advantage, modify a little the application of our method, and so obtain a rule more easily remembered and used. Draw the tangent M B N to the beam at B. It is evident that N C : M A = B C : B A. Then, from the preceding reasoning, make

$$\frac{\text{area } C'P'D' \,.\, dl - K'D'B' \,.\, ck}{\text{area } K'B'G' \,.\, bh - A'F'G' \,.\, ag} = \frac{BC}{BA}.$$

The deflection M A being on the opposite side of the tangent from N C, the similar areas in the above proportion are taken with the opposite signs; that is, K′ D′ B′ being taken minus in the first term, K′ B′ G′ is taken plus in the second, and so of the others. Or we may consider the distances to the right of B plus, and those to the left minus.

It is evident that there is but one position of B′ which will satisfy the condition; for, if B′ is carried still further below K′, the first term of the proportion is diminished and the second term is increased, while, if B′ is raised, the reverse takes place.

26. Another demonstration of the above theorem, which

is brief and which may be more satisfactory to some mathematical minds, is as follows :—

Let M denote the bending moment at any point of a beam, supported in any way ; let E denote the modulus of elasticity of the material and I the moment of inertia of the cross-section. Let the originally straight horizontal line of the beam be the axis of x, and let y be measured vertically. M will be a function of x. Let r = radius of curvature at any point. Then we may write the well-known equation for the curvature

$$\frac{1}{r} = \frac{d^2y}{dx^2} = \frac{M}{EI}.$$

If we integrate this expression once, considering I constant, we have

$$\frac{dy}{dx} = \frac{1}{EI}\int M\,dx = \text{tan. inclin.} = \text{inclination when small.}$$

If we could determine the constant of integration, we could find the inclination of the beam to the horizon at each point. But, if we integrate from 0 to x, the origin being taken at one of the points of support, say C, Fig. 10, we get a complete integral,—the area included between the moment curve and the straight line,—but one expressing only the change of inclination from the inclination already existing at C, or the inclination of the tangent at any point to the line C L.

Integrating again, we have

$$y = \frac{1}{EI}\int\int M\,dx^2 = \text{deflection.}$$

This integral is a volume, and, taken between limits as before, is the summation of each area from 0 to x, into a height dx, giving a cone with a base $=\int_0^x \mathrm{M}\, dx$ and a height x; or it is otherwise equal to the area $\int \mathrm{M}\, dx$ multiplied by the distance of its centre of gravity from the point whose abscissa is x. I is here considered constant, and may be so taken in most trusses. If I varies, it will be expressed in terms of x and introduced within the integral sign.

27. The areas are readily measured by scaling the successive ordinates and multiplying by the constant distance between two ordinates, as is done in calculating the contents of any irregular area by offsets. If the verticals are not too far apart, the areas are practically parabolic segments, triangles, and combinations of the two. The centre of gravity of a parabolic segment, such as $C' D' P'$, is at d, half way horizontally between C' and D'. The centre of gravity of any triangle whose base is vertical, is on a vertical line one-third of the horizontal distance from the base to the vertex. If we connect G' and K' by a straight line, the area $G' K' B'$ will be found to be the difference between a triangle and a parabolic segment, and the moments of these areas about $A' h$ may be computed separately. Similarly, if an area is partly bounded by portions of two different parabolas, the common point of the two parabolas may be connected with the extreme points of the area, and it will be thus divided into a triangle and two parabolic segments. It may be unnecessary

to remark that any area may be divided into a number of parts, the respective centres of gravity found, and then the area moments of these parts calculated and combined, with the same result as if the area had been treated as a whole.

28. If the two spans are equal the *ratio* of the *area moments* becomes an *equality*.

When we have finally determined the position of A′ B′ and C′ B′ for the given load and distribution of the same, we have, as in single spans, the bending moment at each point by taking the proper ordinate and multiplying by H from the stress diagram. Now, upon drawing from 0 two lines, 04 and 05, parallel to B′ C′ and A′ B′, we shall divide the load line into three portions which are: 1–4 the supporting force at C, 4–5 the supporting force at B, and 5–2 the supporting force at A. The beam, as now loaded, has two points of contra-flexure, at D and G. It may happen that, when one of the spans is much longer and more heavily loaded than the other, the point of contra-flexure, G′, on the shorter span, moving towards the outer end, may finally pass off altogether. As G′ moves towards A′, the point 5 will approach 3, and, when G′ reaches A′ and disappears, 5 will pass beyond 3. There will still be some slight pressure on the abutment, although the span A B will be convex upward throughout its whole extent; and the end of the beam will not rise from the abutment A until it is found necessary, in order to satisfy the condition of proportionality of *area moments*, to

so draw A′ B′ that its parallel, 0–5, passes entirely outside of the end 2 of the load line. As soon as this occurs, the beam must be treated as one resting on two supports and overhanging at one end, § 20, Fig. 7.

Finally, lay off P_1 and P_3, at *m n* and *t s*, draw the inclined lines, as was done in Fig. 4, steeper where the load is more intense, and, finding that $p\,q + p\,r = P_2$, we know that our shear diagram is complete.* Then the ordinates between *m t* and the lines just drawn give the shearing forces at all points.

If the load shifts its position, we may draw a new moment curve and then complete the diagram ; but we shall presently show that a law holds for two continuous spans similar to the one advanced for a single span.

29. Before proceeding farther we will show that it is possible, without extra work, to determine the position of B′ and eliminate approximations. We will illustrate by a simple example ; the proof made use of can be extended to any case. Let a beam of two spans, *c* and *d*, Fig. 11, have a single load on each span. The moment curve will be similar to A C D B, the one represented. Let us suppose, for an instant, that there is no bending moment over the pier. Then drawing A I and I B we should complete our figure, and, calling the area of the triangle A C I = **A**, of I D B = **B**, and the distances of the centres of gravity of these triangles, from the verticals

* Perhaps it would have been better to lay off *t s* below *t*.

through A and B, respectively **a** and **b**, we ought to have the proportion $\frac{-\mathbf{A\,a}}{\mathbf{B\,b}}=\frac{c}{d}$. Since a bending moment over the pier does exist, this equation will not be true. Then change the lines A I and I B to A E and E B, moving on the vertical a distance I E $=x$.

We now have the area moments on one side proportioned to the area moments on the other, as c to d. But the area moments on the left are equivalent to

$$\mathbf{A\,a}-\mathrm{AIE}\cdot\frac{2}{3}c,$$

$\frac{2}{3}c$ being the distance of the centre of gravity of A I E from the abutment vertical. Also the area of A I E is $\frac{cx}{2}$. A similar relation exists on the right. Therefore we may state our proportion as follows:

$$\frac{-\mathbf{A\,a}+\frac{cx}{2}\cdot\frac{2}{3}c}{\mathbf{B\,b}-\frac{dx}{2}\cdot\frac{2}{3}d}=\frac{c}{d}.$$

Everything here being known except x, we obtain the distance which B′ (Fig. 10,) should be below K′, when c and d denote the spans, **A** and **B** the areas A′ F′ K′ and K′ P′ C′, and **a** and **b** the lever arms of their respective centres of gravity,

$$x=\frac{3\,(\mathbf{A\,a}d+\mathbf{B\,b}c)}{cd\,(c+d)}=\frac{3}{c+d}\left(\frac{\mathbf{A\,a}}{c}+\frac{\mathbf{B\,b}}{d}\right).$$

If $c=d$, $$x=\frac{3}{2c^2}(\mathbf{A\,a}+\mathbf{B\,b}).$$

Thus we can readily determine the position of B′, and thence the rest of the construction follows simply.

There is no failure when the beam happens to be horizontal over the pier B, for then M A and N C are each zero, and therefore area C′ P′ D′. $d\,l =$ area K′ B′ D′. $c\,k$, or K′ B′ G′. $b\,h =$ A′ F′ G′. $a\,g$.

If great accuracy is not required, and, in any case, after a little practice, one can determine the positions of the centres of gravity, and hence the area moments, without dividing the areas into simpler portions.

The curves of the two spans might be constructed separately, as we should do for detached spans; but, in this case, we must have the same value of H for both curves, and they must coincide on the vertical through the centre pier. Therefore, in drawing the curves, we could start from K′, or any other point on the centre vertical, and work each way. Indeed, the curves might cross the horizontal line through C′, and can be transferred, at any time, if desired, to that line, by measuring off ordinates, either above or below it, so that A′ B′ and B′ C′ shall coincide with or become the horizontal line. Most authors who show any moment curves for girders draw them in this way. The construction of the curve for each girder by a separate stress diagram will be shown presently, in an example of four continuous spans.

30. If we are dealing with a continuous girder of two spans, we can find the bending moment over the centre pier, when each span is loaded *throughout its whole extent,*

with either the same or a different weight per foot run, by means of what is known as "Clapeyron's Formula," or the "Theorem of the Three Moments." This formula, as applied to two spans, resting on abutments at their extreme ends, but continuous over the pier, is—the bending moment

$$M = \frac{w_1 l_1^3 + w_2 l_2^3}{8\,(l_1 + l_2)} = \frac{W_1 l_1^2 + W_2 l_2^2}{8\,(l_1 + l_2)};$$

in which w denotes the weight per foot run, l the span in feet, W the weight of span and load, and the subscript numbers apply to the two spans. Dividing M by H from the stress diagram, in the same units of weight in which w was taken, we shall have the length in feet of the ordinate below the moment curve at the vertical from the pier. This formula only applies when *each span is uniformly loaded.*

31. Let us now make practical application of this method to a truss of two spans, one of 80 feet and the other of 100 feet, represented in Fig. 12, continuous over the pier. Let each truss weigh $2\frac{1}{2}$ tons per panel of ten feet, or 500 lbs. per foot, and let the rolling load be 1,000 lbs. per foot of one truss, or one ton per foot for the bridge. A panel weight for one truss will be, maximum $7\frac{1}{2}$ tons, minimum $2\frac{1}{2}$ tons.

Take 1–2 = 135 tons, the weight of both spans when fully loaded; divide 1–2 into portions of $7\frac{1}{2}$ tons, with end portions at 1 and 2 of $3\frac{3}{4}$ tons at A and C. Take a point 0, preferably opposite the middle of 1–2, and at a distance

in this figure of 50 tons. Since the truss is drawn as $12\frac{1}{2}$ feet high, it will simply be necessary to multiply the ordinates to the moment curves by *four*, to obtain the stresses on the chords, and this can be done, without multiplication, by measuring the ordinates by the proper scale. Leave out the end portions of $3\frac{3}{4}$ tons which come directly upon the abutments, and, commencing at A′, draw the moment curve A′ M C′ parallel to the several lines connecting 0 with the points of division of the load line. Only the extreme lines radiating from 0 are drawn, as the remainder would only confuse the figure and tend to render the point 0 uncertain. Now calculate M B′, either by the formula for determining x or by Clapeyron's formula, by which we have

$$MB' = \frac{M}{50} = \frac{\frac{3}{4} \,.\, 80^3 + \frac{3}{4} \,.\, 100^3}{50 \,.\, 8\,(80 + 100)} = 15.75 \text{ ft.}$$

Draw A′ B′ and B′ C′. The figure A′ M C′ B′ encloses the ordinates for bending moment when both spans are fully loaded. Draw 0–3 and 0–4 parallel to B′ C′ and B′ A′; lay off 2–3 at $c\,d$ and 1–4 at $a\,e$; draw df and eg at an inclination of $7\frac{1}{2}$ tons to a panel, and the figure $c\,d\,f\,g\,e\,a\,b\,c$ encloses the ordinates for shearing force when both spans are fully loaded. The supporting force at the pier $= bf + bg$. The detailed reasons for these constructions can be readily deduced from the case, before given, of a single span.

Next, remove all of the rolling load from A B, including, in this figure, the load on B; the load line will extend

from 2 to 5, and the extreme radiating lines are also drawn here. Use the moment curve C′ M, and add the part from M to A″. Find M B″ as M B′ was found, simply changing $\frac{3}{4}.80^3$ to $\frac{1}{4}.80^3$, and draw A″ B″ and B″ C′. Find anew the supporting forces, and lay off *c h* and *a k*; draw *h i* at an inclination of $7\frac{1}{2}$ tons to a panel, and *k l* at an inclination of $2\frac{1}{2}$ tons to a panel, completing the shearing diagram for this position of the load.

Next, remove the remaining portion of the rolling load from the span B C; and now take the portion of the load line from 5 to 6. As the part A″ M of the moment curve applies to the first span, add the portion M C″, and, finding B^{IV}, complete the figure. M B^{IV} will be, in this case, exactly one-third of M B′. The lines limiting the ordinates in the diagram of shearing force will be *m n* and *p q*. We also take the curve A′ M C″, which represents the case of the shorter span covered with the rolling load, not including the point B, and the longer span unloaded, and, determining A′ B‴ and B‴ C″, find the supporting forces and thence the lines *r s* and *t u*.

Each one of the moment curves might have been drawn independently; but, by the method here carried out, of passing them all through the point M, two complete curves have sufficed for four different positions of moving load; and, in any case of girders with horizontal chords, these two curves are sufficient. The two curves might have passed through M without coinciding for one panel as in this figure, in which case the point B would have been relieved of one half of its rolling

load when one truss was unloaded, and some readers may prefer such a treatment. We should then require a new spacing of the load line.

The moment curve M D′ on the right and the dotted lines for shearing force apply to the particular cases where the span from C to D has upon it no moving load, the load for one set of lines covering B D alone, and for the other set extending from A to D. In the first case the required point on the centre vertical was found to be just below B‴, as drawn, and in the second case a little below B″. The light dotted lines from A″ to M, C′ to M, D′ to M, and from M and D′ to E, show the different areas used in finding the different values of x. In the diagram for shearing force, $y\,v\,w$ and the dotted line between $q\,p$ and $k\,l$ are the limiting lines for the ordinates for a moving load over B D alone, while $x\,z\,g$ and the dotted line between $t\,u$ and $e\,g$ determine the shearing ordinates for a moving load over A D.

32. A little study of the curves for bending moment, both those drawn for the partial and those for the complete load, will show that, for the given intensities of load, all the possible curves, if drawn to pass through M, will lie between M A′ and M A″ on the one side and M C′ and M C″ on the other: also, that the maximum bending moment which tends to make the truss concave upwards occurs when one span is fully loaded, the other being at the same time without travelling load. This fact might be anticipated, since the addition of a load on one span

will tend to diminish the deflection of points on the other. Since the worst bending moment over the pier must occur when both spans carry their heaviest loads, it is evident that M B′ is the longest ordinate for the point B ; conversely, M B$^{\text{IV}}$ must be the shortest ordinate, as the curve to which it relates represents the lightest possible load. All values of x will therefore lie between M B′ and M B$^{\text{IV}}$.

As A′ M C′ and A″ M C″ are the limiting curves, we have also the extreme deviations of the points of contra-flexure, and, from the positions of these points, can readily determine the portion of each chord subject to tension alone, to compression alone, and the portion which has to be designed to withstand both stresses, as the load shifts its position. It will be noticed that the point of contra-flexure for the 100 ft. span shifts from the second panel from B, when only that span carries the moving load, to the fourth panel from B, when only the other span is fully loaded ; while, for the 80 ft. span, the point of contra-flexure shifts from the second panel from B to the seventh panel from B under similar variations of load. The influence of the longer span on the shorter is very marked, as, when the longer one is fully loaded, the unloaded span presses at A with only the weight represented by $a\,k$. A check on the accuracy of construction is found in the fact that, when both spans are loaded and both unloaded, the points of contra-flexure occur at the same place. For a truss with parallel chords we require simply the curves A′ M C′ and A″ M C″, combined, as has been

done here, with the lines which meet at B', B'', B''' and B^{IV}.

33. Let us now discuss the diagram for shearing force. In the first place, the dotted lines in the stress diagram are the ones drawn parallel to those diverging from the pier vertical in the moment diagram, and thus we determined the points marked k, q, e, t, r, m, x, etc.

We shall get the most pressure on the abutment C when B C is loaded and A B is not; the supporting force is then $c\,h$, and the shear on the two spans will be given by ordinates, at the *middle* of each panel, to the lines $h\,i$ and $l\,k$. If the load extends over both spans, the supporting force falls to $c\,d$, and we determine the shear by drawing $d\,f$ and $e\,g$. If neither span has any rolling load upon it, we find the pressure at C to be $c\,m$, and we then draw $m\,n$ and $p\,q$; finally, if A B alone carries the rolling load, the pressure at C becomes $c\,r$, and the shearing diagram will be completed by the lines $r\,s$ and $t\,u$.

Again: if the rolling load, at first extending entirely from A to C, moves off from the portion C D, the supporting force at C diminishes from $c\,d$ to $c\,x$, and the shear will be given by ordinates to the lines $x\,z\,g$, etc., the point z occurring in the first lightly loaded panel D. If the rolling load covers B D, but the part on A B is supposed to be removed, the supporting force at C will immediately increase to $c\,y$, and the bounding lines for the ordinates will now be $y\,v\,w$, etc. As it is manifest that $h\,i$ and $m\,n$ are the limiting lines for the extreme cases of load over

B C alone and load over neither span, and as the load may cover any number of panels, from one to ten, from B towards C, the lines similar to *y v w*, for the different positions of the load, will shift between *h i* and *m n*, and the point *v* will move on a curve, found to be a parabola, from the middle of one end panel to the middle of the other in the span B C. The ordinate to this curve will give the maximum possible shear in any panel for a load advancing from B. At the same time the line for the unloaded span moves between *k l* and *p q*. Draw the parabola *n v h* by § 10, commencing in the middle of the end panels.

If a load had advanced from A, over A B, and then extended from B towards C, we should have obtained, by exactly the same reasoning, a parabola, traced by the point *z*, on the tangents *d f* and *r s*. As the ordinates to this parabola, above the line *b c*, are less than those to *h v n*, the latter only need be considered.

If a rolling load advances from C, the span A B being unloaded, the pressure at C increases from *c m* to *c h*, and the different limiting lines for shear will move between *h i* and *m n*; but, as the inclinations of the lines which correspond to *y v* and *v w* will be reversed, since C D now is loaded and B D is not, the point *j* at the angle will, like *v*, trace a parabola, from the middle of the panel near *m* to the one at *i*, of the opposite curvature. If, at the same time, A B is loaded and a load advances from C towards B, we find the parabola drawn in the figure from *r* to *f*. As this one includes the former, it is the only one needed.

While the load is shifted as above, the line in the other span vibrates between the parallels *k l* and *p q* in the one case, or *t u* and *e g* in the other.

34. The two parabolas *h v n* and *r j f* then limit all the ordinates for shearing force in this span, for every position of moving load, the portion of the parabola *h v n* which lies above *b c* giving the maximum shearing stress upwards, and requiring diagonals sloping one way, depending upon whether the truss has ties or struts for diagonals, and the part of *r j f* which lies below *b c* giving the maximum shearing stress downwards, and requiring braces sloping in the opposite direction. Upwards and downwards are relative terms, and are used on the supposition that the shear is taken on the right of the plane of section as shown in Fig. 6, enlarged view of panel N O Q P. A similar construction supplies the required parabolas for the span A B, and thus all possible positions of the travelling load are provided for. It will be noticed how the counterbracing is shifted from the middle of the spans towards the free ends.

The desire to have the diagrams clear, while they are on so small a scale, forbids the drawing of moment curves and lines of shear for various positions of the load ; but, if the reader will construct a figure for himself, with the load shifted, panel by panel, for a few panels, he will perhaps more readily see the truth of the statements here made as to the limiting values of bending moment and shearing force. Our object has been to set forth the

method to be pursued in designing a series of trusses by diagrams, and to explain sufficiently at length to enable any one using the method to satisfy himself of its applicability.

The two tangents $m n$ and $f d$ must intersect on $b c$, as they apply to the cases of uniform loads over both spans, and the usual relations between maxima and minima bending moments and shearing forces must also hold good. The different parts of the chords under different stresses, determined by the limiting positions of the points of contra-flexure, are shown in the figure, the portions always in tension being drawn with fine lines, the portions always in compression being represented by heavy lines, and those portions liable to undergo both stresses having double lines.

35. Remember, that, while the full lines in this figure show all of the curves and diagrams required for a full discussion of a continuous girder with horizontal chords, a truss with curved or inclined chords necessitates the use of a moment curve for each position of the load, to be combined with the shearing diagram, as explained in § 15. Also, Clapeyron's formula can be applied only when each span has some *uniform load throughout its extent;* when the load is partial or is irregularly distributed, find x by § 29. Find the stresses on the diagonals and verticals by § 11 or § 15, as the case may be.

If the two spans are equal, the number of lines becomes less, as B$''$ and B$'''$ will coincide, and only the shearing diagram for one span need be drawn.

CHAPTER IV.

CONTINUOUS GIRDER OF MANY SPANS.

36. This graphical method is perfectly applicable to continuous girders, having any number of spans, of any lengths, and loaded in any manner. The example which we have chosen, see Fig. 13, shows four spans, of successively 80, 100, 50 and 40 feet, loaded as represented, with a load of the same intensity as given in the previous example, viz.:—from L to M and N to P, travelling load of five tons per panel, and steady load throughout, from A to E, of two and one half tons per panel.

To prevent the load line and stress diagram from occupying too much space, as well as to show the perfect practicability of drawing separate moment curves for each span, as we should do for single trusses, we have laid off the load lines of the longer spans independently, but have taken the same value for H in all of the diagrams, and this *must* be done. The stress diagram for the 80 ft. span is 012; the marks of division show the weights on the respective panel joints. Starting from A′, draw the moment curve for this span, terminating at B′. From B′ draw the moment curve for the 100 ft. span, by lines parallel to those which would complete the stress diagram 0′ 34, 3–4 being the load on this span, the curve ending at C′. As the last two spans are short, draw 5–6 equal to

the load on both spans, and then construct C′ D′ E′, as has been done in previous examples. Now draw A′ B′, B′ C′, C′ D′ and D′ E′. Compute the areas between each of these lines and the respective moment curves, and determine the position of the centre of gravity of each area. Those areas which are not simple parabolic segments can be divided, as explained in § 27.

37. Now we have to determine, in the same way as we found x for two spans, the distances x, y and z, or B′ F, C′ G and D′ I, the ordinates at the piers, required to complete the diagram for bending moments. Let the spans, commencing with A B, be l_a, l_b, l_c, l_d. Let the areas be represented by **A**, **B**, **C** and **D**. Let the distance of the centre of gravity of **A** horizontally from A be **a**, and from B be **a**′; let **b** and **b**′, **c** and **c**′, **d** and **d**′ denote the similar distances for **B**, **C** and **D** from the verticals on their left and right.

Then, considering the two spans A B and B C, we must have, (see § 29,)

$$\frac{\mathbf{Aa} - \frac{x}{2}\cdot\frac{2}{3}\,l_a^2}{\mathrm{B'FC'}\cdot\frac{2}{3}\,l_b + \mathrm{FC'G}\cdot\frac{1}{3}\,l_b - \mathbf{Bb'}} \quad\text{or}\quad \frac{\mathbf{Aa} - \frac{x}{3}\,l_a^2}{\frac{2x+y}{6}\,l_b^2 - \mathbf{Bb'}} = \frac{l_a}{l_b} \qquad (1.)$$

$$\frac{\mathbf{Bb} - \frac{x}{2}\cdot\frac{1}{3}\,l_b^2 - \frac{y}{2}\cdot\frac{2}{3}\,l_b^2}{\frac{y}{2}\cdot\frac{2}{3}\,l_c^2 + \frac{z}{2}\cdot\frac{1}{3}\,l_c^2 - \mathbf{Cc'}} \quad\text{or}\quad \frac{\mathbf{Bb} - \frac{x+2y}{6}\,l_b^2}{\frac{2y+z}{6}\,l_c^2 - \mathbf{Cc'}} = \frac{l_b}{l_c} \qquad (2.)$$

$$\frac{\mathbf{Cc} - \frac{y}{2}\cdot\frac{1}{3}\,l_c^2 - \frac{z}{2}\cdot\frac{2}{3}\,l_c^2}{\frac{z}{2}\cdot\frac{2}{3}\,l_d^2 - \mathbf{Dd'}} \quad\text{or}\quad \frac{\mathbf{Cc} - \frac{y+2z}{6}\,l_c^2}{\frac{z}{3}\,l_d^2 - \mathbf{Dd'}} = \frac{l_c}{l_d} \qquad (3.)$$

Equations (1.) (2.) and (3.) may be reduced to

$$(l_a + l_b)\,x + \frac{l_b}{2}\,y = 3\left(\frac{\mathrm{A\,a}}{l_a} + \frac{\mathrm{B\,b'}}{l_b}\right) \qquad (4.)$$

$$\frac{l_b}{2}\,x + (l_b + l_c)\,y + \frac{l_c}{2}\,z = 3\left(\frac{\mathrm{B\,b}}{l_b} + \frac{\mathrm{C\,c'}}{l_c}\right) \qquad (5.)$$

$$\frac{l_c}{2}\,y + (l_c + l_d)\,z = 3\left(\frac{\mathrm{C\,c}}{l_c} + \frac{\mathrm{D\,d'}}{l_d}\right) \qquad (6.)$$

These equations may be extended to any number of spans by commencing with (4.), ending with one like (6.), and writing similar equations to (5.) a sufficient number of times to make in all one less equations than the number of spans. If $l_a = l_b = l_c$, etc., the equations simplify a little.

To show that no difficulty exists in the solution of the above equations, apply them to the above case, and, since the second members are known quantities, denote them by P, Q and R. Then

$$\begin{array}{lll}
180\,x + 50\,y = \mathrm{P} & (7.) & \\
50\,x + 150\,y + 25\,z = \mathrm{Q} & (8.) & \\
25\,y + 90\,z = \mathrm{R}. & (9.) & \text{Multiply (8) by 3.6} \\
\hline
180\,x + 540\,y + 90\,z = 3.6\,\mathrm{Q}. & & \text{Subtract (9.)} \\
\hline
180\,x + 515\,y = 3.6\,\mathrm{Q} - \mathrm{R}. & & \text{Subtract (7.)} \\
\hline
465\,y = 3.6\,\mathrm{Q} - \mathrm{R} - \mathrm{P} & & \\
y = \dfrac{3.6\,\mathrm{Q} - \mathrm{R} - \mathrm{P}}{465}. & &
\end{array}$$

Substitute in (7.) and find x; then in (9.) and find z.

38. In the practical solution of this example, with the given load distributed as shown in Fig. 13, we meet with

one peculiar result: the value of z is found to be *minus*, that is D′I must be measured off from D′ upwards. This fact shows that there is no point of contra-flexure on the forty feet span, and no negative moment over the pier D. The load on the 100 ft. span has a tendency to lift the bridge off from the pier at D, and consequently the pressure there will be found to be small. If the point I had come on the straight line joining G and E′, the bridge would have been lifted entirely clear of the pier D, and, if I had come above the line GE′, another solution would have been required with the pier D considered as removed, making CE one span, or else the truss would have required bolting down at D.

39. Having drawn A′ F, F G, G I and I E′, draw, in the respective stress diagrams, lines 0 7, 0′ 8, 0″ 9 and 0″ 10, parallel to them, cutting off the supporting forces on the several piers arising from each truss, and then complete the diagrams for shearing force. We shall find that, by drawing the stress diagrams separately, as has here been done, we get the supporting force on any pier, carrying the ends of two spans, in two portions, one belonging to each span; and hence the point of termination of the line for shearing force on the vertical through the pier is at once known. Thus 1–7 equals ap and 7–2 gives bf, 3–8 gives bm and 8–4 determines cg. The pressure on B is $bf + bm$. Notice that, as di and dk come on opposite sides of d, the pressure on D, with this load, is $dk - di$.

40. If a continuous girder of several spans is to be completely discussed, Clapeyron's formula may be usefully applied to the determination of bending moments over the piers, subject always to the restriction of uniformity of load; as before remarked. Denoting the bending moments at successive points of support by M_0, M_1, M_2, etc.; the spans by l_1, l_2, l_3, etc., in feet, and the respective loads per foot by w_1, w_2, w_3, etc., we may write this formula:

$$M_0 l_1 + 2M_1(l_1 + l_2) + M_2 l_2 = \tfrac{1}{4}(w_1 l_1^3 + w_2 l_2^3),$$
$$M_1 l_2 + 2M_2(l_2 + l_3) + M_3 l_3 = \tfrac{1}{4}(w_2 l_2^3 + w_3 l_3^3),$$
$$\text{etc.,} \qquad\qquad \text{etc.}$$

We have in all one less equations than there are spans. As the ends of the girder are usually free, there will be no bending moments at the two abutments, or, for n spans, M_0 and M_n will equal zero, and there will be left $n - 1$ unknown bending moments to be determined from $n - 1$ equations.

CHAPTER V.

PIVOT BRIDGES OR DRAW SPANS.

41. Having shown the method of treatment for the fixed spans of a bridge, either continuous or discontinuous over the piers, we wish to complete the investigation by discussing what is called the *draw span*. This span is capable of being turned horizontally on a pivot at its middle, so as to open two channels, one on either side of the centre pier, for the passage of vessels. When it is open, it acts as two beams or trusses, each fixed and horizontal at one end, with a uniform load, arising from its own weight, over its whole extent. The two portions are usually, though not necessarily, equal in length, and they balance on the pivot and turntable. When the draw is closed, it may be elevated by cams or wedges under the free ends so as to bring a greater or less pressure upon those piers, or it may simply swing over its wall plates or seats, without practically pressing on the piers until a travelling load comes upon it, or finally it may be secured by horizontal locking bolts, so arranged that, while they do not bring the draw any more closely in contact with the wall plates, they prevent one end from rising from its seat when the travelling load first comes upon the other end.

42. We have, then, several cases with which to deal.

1*st*. If the pivot or draw span is supported at its ends, when closed, so that the moment over the centre pier is the same as exists in a continuous girder of two equal spans, (in which case it is necessary that each end support shall carry, when there is no travelling load on the draw, *three eighths* of the weight of *one* span,) the draw, when closed, will be circumstanced precisely like a continuous girder of two equal spans. To obtain the required amount of supporting force it is necessary to provide lifting cams with powerful leverage, or to use hydraulic jacks. If then we draw our diagrams for two equal continuous spans as described before, we have only to add the moment curve and diagram for shearing force for the draw *open*. A consideration of the method given for a beam overhanging at one end, will show the construction in this case. Here the beam overhangs at both ends, and the two supports have become united in one, the beam balancing upon it.

43. Construct, if it has not already been done, the moment curve for the truss loaded with its own weight. Remember that the *end joints* each carry one-half of a panel weight. Therefore, from the points where the moment curve cuts the verticals from the extreme ends of the draw, draw lines parallel to lines in the stress diagram from 0 to the extreme ends of the load line, and these lines must cut the vertical from the centre pier in the same point. By reference to Fig. 14, we can see that the draw when open is in the condition of the beam A B C, if

the abutments A and B are removed. The moment curve will be A′E′C′, and A′B′ and B′C′ are tangents to the curve at its extremities. The ordinates to the moment curve, thus intercepted, will be, when multiplied by H, the bending moments at the joints.

Draw lines from the extremities *a* and *c* of the base line for shearing force to a point *b*, on the centre vertical, below the base line a distance equal to the weight of one span, or one-half of the load line, and the ordinates at the middle of the panels will show the shearing force when the draw is open.

The diagrams for this case are thus completed; that is, these lines just described are to be added to the curves, etc., required for two equal continuous spans. When the draw is open, there will be tension throughout the top chord and compression throughout the bottom chord, and braces will be required, all sloping one way, from the centre to the end. The stresses when the draw is closed, and the direction of the braces required, will be as stated in §§ 32, 34. If the supporting forces at the ends exceed or fall short of the amount before stated, three eighths of the weight of one span, the case requires a different treatment. From the power applied to the cam or jack the actual amount of the supporting force may be approximated to, and the new state of affairs readily be classed under a case soon to be taken up.

44. 2*d*. A draw, as usually constructed, is screwed up at the centre, until almost the entire weight and bearing is

upon the pivot, and the ends of the truss scarcely do more than touch the bearings on which they close, as is shown by the immediate tilting up of one end of a draw when a train enters the other end. The truss may then practically be considered as poised on the centre when shut and unloaded, with the same stresses as when open. Let A B C, Fig. 14, represent the draw, closed, poised on the pivot B, and barely in contact with the abutments at A and C. The curvature in the figure is exaggerated; but every draw is practically curved in this way when supported in the middle and deflected under bending moments arising from its own weight, even when it is actually straight and horizontal, as is proved by the existing tension in the top chord and compression in the bottom chord; and the points A and C can be brought to a level with B only by a *camber* previously induced in the truss.

When a rolling load passes on at D, the farther end F rises, the truss is carried by two supports, with one end overhanging, and the construction of the diagrams will follow the method described in § 20. As described in § 43, A′ E′ C′ B′ is the diagram for bending moment of A B C, the draw open, and also for the draw closed but not raised at the ends by cams. By drawing T′ D′ as the new portion of the curve required by the load T D, we have D′ T′ E′ C′ as the new moment curve. Any load on D E will not alter the bending moment on E F, and therefore C′ B′ is still one of the lines, and B′ D′ must be the other required to complete this diagram. The point of contra-flexure near T′ is readily seen. A line parallel, in

the stress diagram (not shown in this figure), to D′B′, will give us *a d*, the supporting force at D, and the lines *d t e* with *b c* will determine the ordinates for shearing force.

45. The further end of the draw will rise still higher, as the load advances over the first span, and finally reaches the centre pier. The further progress of the load will cause F to move down again, but it will not come down to the bearing on G, until the train or other moving load has advanced a certain, often a considerable, distance over the free span, sometimes more than one-fourth of the space from the centre pier. The important point to be determined is the position, S, of the front of the rolling load on I K L, when the end, L, of the beam is just forced down to its abutment.

Draw through K a tangent Q K M to the beam at that point. When the beam rests on the abutments the deflections at the ends from the tangent at K are M L and Q I. From the similar triangles M N L and P Q I, made by vertical lines through I and L and lines perpendicular to Q M, any ratio which exists between M L and Q I will hold between N L and P I. If we add I R to P I, and subtract its equal, L O, from N L, we have an entirely similar proportion to the one deduced for a two-span continuous truss, namely:

$$NO : PR = KN : KP = KO : KR = 1, \text{ or } NO = PR.$$

Now M L is proportional to the *area moment* on the

span K L, as explained in §§ 24, 25 ; Q I is proportional to the area moment of the span K I, and L O, = I R, is proportional to the original area moment of either span, before any rolling load has come upon it, or to the area C′ B′ E′ multiplied by the distance of its centre of gravity from the vertical through L. Call this area moment **C c**.

46. The curve I′ E′ L′ being drawn, the condition that the end L shall rest upon the abutment will, therefore, be satisfied when the area moment to the left of E′ K′ plus **C c** equals the area moment to the right of E′ K′ minus **C c**. This equation will determine the position of K′, as in former cases. But note that, when we find a point K′, satisfying the above condition, so placed that a tangent to the curve at L′, or a line parallel to the last radiating line in the stress diagram to the *extremity* of the line of loads, cuts E′ K′ above K′, the span has risen from the abutment. For it is necessary that a parallel to K′ L′ should cut off some portion of the load line to give any supporting pressure to L. When one end of the span is off the abutment, see § 44.

To find the distance E′ K′ or x for every position of the end of the travelling load between S and L:—draw a straight line from I′ to E′, and one from E′ to L′ ; call the areas between these lines and the moment curve respectively **A** and **B**, **A** being the area belonging to the side which is entirely loaded, and the distance of their centres of gravity, horizontally from I′ and L′, **a** and **b**. Then, by the same course of reasoning given in § 29, Fig. 11,

and from the equation stated above, we have, letting l equal either span,

$$\mathbf{A a} - \frac{x l^2}{3} + \mathbf{C c} = \frac{x l^2}{3} - \mathbf{B b} - \mathbf{C c}.$$

$$x = \frac{3}{2 l^2} (\mathbf{A a} + \mathbf{B b} + 2 \mathbf{C c}).$$

47. If the curves are drawn as usual in these pages, I′ E′ being common to all of the curves for movements of S, the head of the load, towards L, we have $\mathbf{A a}$ and $2\,\mathbf{C c}$ constant, as well as the divisor $\frac{3}{2 l^2}$; so that, for each new position of the load, we have only to calculate $\mathbf{B b}$, divide it by $\frac{3}{2 l^2}$, and add the quotient to the previous constant quotient, to obtain x, equal E′ K′.

The shear is then readily obtained; for the load from I to S it is limited by the lines *i k k s c*. When the load extends to L we have the symmetrical curve I′ E′ V′ and the shearing diagram *a u w v c*. The increase of rolling load from S to L diminishes the supporting force at I from *a i* to *a u*, and manifestly the supporting force at I will be a maximum when only the span I K is covered by the travelling load.

The points of contra-flexure will be found nearer the ends for given loads than in a continuous girder of two spans, as might be expected; since the load has to overcome or neutralize first the initial negative moment of flexure. And, consequently, the bending moment over the centre pier is greater than in a continuous girder. Most draw trusses have a greater depth at the centre

pier than at the ends, thus diminishing the stress on the chords at that point below the amount which would exist for parallel chords.

48. It will perhaps be more convenient to show now, before we give the entire diagrams for a pivot bridge or draw span, the modification which is introduced by case

3*d*. Where the ends of the draw are prevented by locking bolts from rising from the abutments, the action will be similar to that produced by hanging an additional, but variable, weight at F, just sufficient to bring F in contact with G. The greatest weight will be required when the rolling load extends from D to E. It is readily seen that, if F must always remain on G, the condition required for case 2, instead of being limited to loads which give a pressure on L, must be satisfied for every position of the load from D to F; that is, we must always have

$$x = \frac{3}{2\,l^2}\,(\mathbf{Aa} + \mathbf{Bb} + 2\,\mathbf{Cc}).$$

This value of x determines K′. Now if the line to be drawn in the stress diagram, parallel to K′ L′, passes beyond the end of the load line, the additional length of load line required to meet it will be the upward pull on the bolt at L, and this pull will increase, commencing with zero at S, until the rolling load retires to K, and will then diminish to zero again when the rolling load entirely moves off at I. This modification makes the only differ-

ence in the treatment of cases 2 and 3. In the shearing diagram, the amount of upward pull on the locking bolt, being opposed in direction to a supporting pressure, is to be laid off from *c*, downwards, and, from its lower extremity, will then be drawn a line parallel to, and taking the place of, *c s b* or *c s k*, etc. This pull affects the stresses on all portions of the draw and shifts the points of contra-flexure.

49. *4th.* Suppose that the draw, instead of being circumstanced as in case 2, is raised at the ends by cams or jacks, but that the supporting forces do not equal those required by case 1, where A and C are on the same level with B. Find, from the known power applied to the cams, the amount of the supporting force at each end when the draw is unloaded. Lay off these amounts, each on the proper end of the load line, and draw lines from the two points of division so obtained to the point in the stress diagram usually marked O. Construct the moment curve for the spans unloaded, and then draw lines, from the extremities of the curve to the centre vertical, parallel to the lines just drawn in the stress diagram. The ordinates so cut off will be the ones required to determine the bending moments on the closed and unloaded draw, and the area moment on one side, (or the moments on each side, if not the same, owing to different supporting forces at the ends,) will represent the quantity to be used, instead of Cc, in cases 2 and 3.

The ends of the draw may be raised so as to give a

pressure of more than three-eighths the weight of one span on each abutment. It will then be necessary to determine the amount and proceed by the steps just described.

50. A general case might be made of the difference of level between the centre pier and the abutments. If A and C are above B, we shall have the last case. If A and C are sufficiently above B, we shall have no pressure on B and hence one single span. If A and C are level with B, Cc will equal zero and we shall have case 1. If A and C are below B, we shall have cases 2 and 3. An equation similar to those which have preceded, formed by the introduction of two terms in Cc, one for each span, into the value for x given in § 29, would probably admit of this general discussion.

51. If we take a draw span, such as is represented in Fig. 15, we may proceed to find the stresses on the different parts, under the supposition that holding down bolts are used; in other words we may apply case 3.

Let the draw be 240 feet long, making two spans of 120 feet each, ($= l$,) divided into twelve panels of 20 feet each; let the height at the centre be 25 feet, and at the ends 20 feet. The dead weight is assumed to be 10 tons to a panel, and the travelling load is also 10 tons to a panel. An increase in the number of panels and an unequal distribution of dead and live load would not have rendered the construction more difficult, but would have confused the figure. In order to have the moment curves

well separated, with the present scale, we have taken H as 60 tons only.

To proceed to the construction: 1–2 represents 240 tons, the maximum load. Take 0 at a distance of 60 tons, opposite the middle of 1–2. Draw, if convenient, A′ M C′, for the draw entirely loaded, by commencing at C′ with a line parallel to 3–0, and ending at A′ with a line parallel to 4–0, 20 tons being carried on each joint, except A and C, which carry 10 tons each. Next, through the point M, draw A″ M C″ for the draw unloaded; it may be convenient to use one of the sides of the polygon already drawn through M, as, for example, the one parallel to 0–7, and we may, therefore, construct each way from this side, as shown in the figure, taking 10 tons on each joint, and terminating at C″ and A″ with lines parallel to 0–8 and 0–9. As five tons, when the draw is free or is open, rest on A and C, draw from A″ and C″ the lines A″ B″ and C″ B″ parallel to 0–5 and 0–6. The ordinates between A″ M C″ and A″ B″ C″ are proportional to the bending moments at different points of the draw open, or closed and unloaded. Calculate the area A″ M B″ = **C**, find the distance of its centre of gravity from the vertical through A, and denote it by **c**. (See § 27. If A″ M may be considered a close approximation to a parabola, $\mathbf{c} = \frac{3}{4}$ A B.)

Draw a straight line from A′ to M and compute the area enclosed between the curve A′ M and the line just drawn; call it **A**; then multiply by the distance **a**, =

$\frac{1}{2}$ A B, of its centre of gravity from A. To find, for the curve A′ M C′, M B′=

$$x = \frac{3}{2\,l^2}\,(\mathbf{Aa} + \mathbf{Bb} + 2\,\mathbf{Cc}),$$

we have **Bb**=**Aa.** Having made this computation, lay off M B′, and draw A′ B′ and C′ B′, determining the bending moment for the draw closed and entirely loaded. Draw a straight line from C″ to M, compute **Bb** for this curve, and, using **Aa** as before, find by the same formula, M B‴, and draw A′ B‴ and C″ B‴, as well as A″ B‴ and C′ B‴, determining the bending moment for the draw when one span is loaded and the other unloaded.

52. Next draw from 0 lines parallel to those which meet at B′, B″ and B‴, and lay off the supporting forces thus found at *a* and *c*. These reactions will be *a e* and *c d* when both spans are loaded, *a k* and *c n* when A B is loaded and B C unloaded, and *c g* and *a l* when the load is upon B C alone. It is plain that the pressure upon the abutment at A will be greater when A B alone is loaded, than when a load over a portion or the whole of B C acts as a partial counterpoise, and the maximum pressure at A,= *a k*, will occur when the whole of A B is loaded. A weight between A and B will tend to raise C from the abutment, and will cause a shearing stress on that locking bolt. This stress will increase as the load on A B increases, until all the weight is added, but, as soon as the load passes beyond B toward C, the stress on the bolt at

C will diminish. Therefore *c n* is the maximum shear on the bolt, and is drawn below *a c* as a negative supporting force, because the line drawn from 0, parallel to B‴ C″, to determine it, strikes *beyond the end* 6 of the load line to that amount. At the other end, 5–10 = *a l*.

Now draw *a h* and *c h*, at an inclination of ten tons to a panel, cutting off *b h*, equal to the weight of B C or A B unloaded; *a h c* will be the shearing diagram for the unloaded draw, either open or closed. Draw *e f* and *d f*, at an inclination of twenty tons to a panel, and *a e f d c* will be the shearing diagram for the draw closed and entirely loaded. Then *k i*, parallel to *e f*, and *n m*, parallel to *c h*, will be the lines required for the diagram when A B is loaded and B C unloaded; while *g i* and *l m* will apply to the reverse case. As will appear later, parabolas drawn on the tangents *l p* and *p f*, *g r* and *r h*, having points of tangency at the middle of the first panels from each point of support, will complete all *necessary* lines in the shearing diagram. To easily construct these parabolas, see § 10.

53. As the draw in this example has an inclined upper chord, we shall need other moment curves. Suppose the load, advancing from A, extends to and includes G. Using the portion C″ M for the lightly loaded side of the draw, add the two sides of the polygon which will complete G′ M C″, compute x for this curve as was done for the others, and draw lines from G′ and C″ to the middle vertical, meeting it at a point a little removed from B″ towards B‴; for the weight added at E and G will in-

crease the bending moment over the point B. The lines for shearing force, (not drawn in the figure,) starting from a point above a, will run to t, and thence, at a less inclination, to a point below m; on the other span the line will lie between $h\,c$ and mn.

Next suppose that, instead of a rolling load from A to G only, we first load B C throughout and then add the load from A to G as before. We must now use the curve $G'MC'$, and the bending moment at the centre pier will be increased beyond the amount existing when only one span is loaded, so that the inclined lines will now meet the centre vertical between B''' and B'. Thus the two lines drawn from G' to the centre vertical are accounted for. Finding the supporting forces under the new condition of things, we shall have, in the diagram for shearing force, the lines aw, wu, uz, and a line between and parallel to ig and fd. We thus find a greater negative shear on A B under this load than under the former.

Add a load on J. Draw $J'M$. The bending moment over the centre pier will be slightly increased, and we can construct, as we did from G', the two lines from J' to the centre vertical, and in the shearing diagram we shall find, when B C also is loaded, the lines $a\,y$, $y\,v$, etc., as before. A load added to L will, in the same way, determine the point s. The curves starting from E', G', J', L' and A' represent the successive increments of rolling load on E, G, J, L and P, and, as combined with MC'' or MC', apply to the cases where B C is unloaded or loaded. All the lines to the centre vertical are not drawn, but enough

of them are shown to explain the action. The letters at the extremities of the curves are intended to indicate the last fully loaded point. A rolling load on B will be found not to alter the bending moment, hence the curve for P becomes the one from A′.

54. As the load advances from A to B then, the span B C being fully loaded, we shall find that the ordinates for the maxima stresses in the successive panels occur immediately in front of the load, and terminate in the points *u*, *v*, *s*, etc., which points will lie in a parabola, described on the tangents *lp* and *pf*, with tangent points in the middle of the first and last panels. If B C were *unloaded*, these ordinates would terminate in a parabola drawn on the tangents *a q* and *q i*, and shown in the figure as a dotted line; but, as the former parabola gives the maxima stresses in this direction, it alone need be drawn. A symmetrical one can be sketched on *f r n*, for a load advancing from the other end, but it will not be necessary.

Similarly, if the load extends from A, past B, to V, we shall have the curve V′ M combined with A′ M, and, if only B V is loaded, the curve V′ M combined with A″ M. A load on B V alone will cause a greater supporting force at C than when A B is also loaded; therefore the increments of load from B towards C, while A B is unloaded, will give us the points on a parabola drawn on *h r* and *r g*. The dotted parabola *d m* on the parallel pair of tangents below, belongs to the case of a load extending from A en-

tirely to the successive points on B C. All other loads moving from A towards C will give less values than those now obtained, and the maxima shears in all of the panels are thus determined by ordinates, at the middle of the panels, from *a b c* to the parabola *l u s f* below, or to that portion of the other parabola *g h* above, *a b c*. Finally, we make the draw span symmetrical, and so provide for moving loads in the other direction.

55. The point of contra-flexure, advancing from the outer end as a load enters upon the draw, will be seen to move no nearer the centre pier than the third panel from the end. Consequently K R will always be in tension, G B will always be in compression, while K D and G A must be designed to withstand either stress. The moment at the joint J, being the last moment which is always the same as that over the pier, will be resisted, if we take J as origin, by J K multiplied by the horizontal projection of the tension in K N, or, if we take K as origin, by J K into the compression in J G. Thus we determine the points K and G of the lengths R K and B G.

The maximum ordinate at any joint being readily selected from the figure A′ M C′ C″ B′ A″, and multiplied by H, if we divide by the height of the truss at that joint, we shall have the *horizontal* stress on that side of the joint not touched by the diagonal in action at the time. The horizontal stress in the inclined chord must be increased in the ratio of the actual length of that portion of the chord to the length of a panel horizontally. A ready

construction is to draw a horizontal line equal in length to the horizontal stress, from one end draw a line parallel to the portion of the inclined chord in question and limit it by a vertical line from the other end. The hypothenuse represents the desired stress on the inclined chord. A curved piece may be treated as straight between the two joints, for finding the direct stress; its curvature introduces a separate bending moment on the piece itself.

56. Finally, to find the stresses on the diagonals and verticals:—compare § 15, etc. Take, for example, the pieces L Q and P Q. The maximum shearing stress in the panel L P will be the ordinate from $a b$ to s. Lay off this ordinate at $o s$ in the lowest figure. This shear will be caused by a rolling load from A to L inclusive, together with one from B to C, (and is to be considered a positive or upward shear, as shown by the arrow in the panel, although the combination of the two spans of the draw brings this parabola below the line $a b$). The curve of moments will therefore be L′ M C′, and the ordinates for bending moment, under that load, at L and P will be L″ N′ and P′ Q′; multiply these ordinates by H, and divide by L N and P Q respectively, thus obtaining the *horizontal* stress from N towards Q and the horizontal stress on P L. Lay off s L equal to the latter, and, drawing o Q parallel to N Q, make the horizontal distance of Q from o equal to the former. L s and o Q are drawn to the left of the vertical $o s$ representing the shear, because the bending moment tends to make the truss convex

upwards; or the reverse of Fig. 6. If no error has been made, L Q when drawn will be parallel to L Q of the truss, and will give the amount of the existing stress, while the line marked P–Q, drawn vertically upwards from the point marked L, will be the stress on the vertical P Q. The remainder of the figure applies to the other pieces of the web, as shown by the letters. We shall thus obtain the stresses on all of the diagonals which incline from the centre pier down towards the abutment. B R will have upon it double the stress shown in the figure, as it resists the action of the inclined pieces in both halves of the draw. There remains the diagonal S T, required by the shear which arises in the panel S C when the span B C alone is fully loaded. The bending moment at S being then of the kind to cause convexity downwards, construct this figure on the right of the vertical at o. There will be no tension on S C, and we are consequently reduced to a triangle, giving S–T and C–T as drawn.

A study of the diagrams of Fig. 6, and the explanations therewith, will show what modifications would be introduced by the inclination of the bottom chord, the substitution of struts for ties, or of a load on the top chord for one on the bottom chord.

57. Without drawing additional diagrams we can probably determine, by inspection of Fig. 15, what changes would be effected in the draw by the removal of the locking bolts, bringing it under case 2, § 44. Both when the draw is entirely loaded, and when the rolling load is alto-

gether removed from the draw, there is no force exerted on the bolts. The absence of the bolt will therefore make no change in the position of B′ and B″. The point B‴, found when one span only is fully loaded, will be situated a little nearer M, owing to the omission of the moment over B caused by the pull on the bolt multiplied by the span B C ; the point of contra-flexure will therefore come somewhat nearer the centre pier, and the extent of each chord subject to but one kind of stress will be diminished. All of the values of x which would give any pull on the bolt, and which correspond to loads on any portion of R S, Fig. 14, will be slightly less.

In the shearing diagram we need to consider only the changes which take place in the two limiting parabolas for maxima values, $l\,v\,s\,f$ and $h\,g$. As $l\,v\,s\,f$ is the curve for a load advancing from A, while the other span is fully loaded, and as, under these circumstances, the holding down bolts cannot be in action, this parabola will not be changed. When B C, and it alone, is covered with a rolling load, the supporting force at C is diminished by the amount of the *pull* $a\,l$ on the abutment at the other end. Remove the bolt at A, and the point g will now be found to have moved farther from d by the amount $a\,l$ or $c\,n$. That is, to find g, lay off $n\,g$ from c. This increase of supporting force at C will affect the shear at all points of the span, when B C is loaded, by just this amount, and therefore $g\,i$ will move parallel with itself to the new distance from c just stated, and $m\,l$ will coincide with $a\,h$. Therefore the

parabola $h g$ will rise to its new tangents, increasing the shear in panels near C.

The coincidence of $m l$ with $a h$, or its disappearance, will not affect the position of $l v s f$, but we shall have to draw $l m$ where it stands in this figure, to find this parabola. With these suggestions no difficulty need be experienced in making the diagrams for case 2.

CONCLUSION.

It will be noticed that we have not, in what has gone before, attempted to give different forms of trussing. We have rather endeavored to set forth the method plainly, and perhaps have been, for that reason, at times diffuse; a clear understanding of the line of procedure will enable one to apply it to any type of bridge desired. Where compound systems of trussing are to be used, it will probably be advisable to draw separate diagrams for the simple systems, and then combine the results. We have also, in the bridges used as examples, taken both the steady and rolling loads as uniformly distributed over the proper portions of the span. Any one who wishes to make the load heavier in one portion than in another, as is sometimes done when we consider the weight of a locomotive aside from the average weight of a train per foot, can readily change the divisions on the load line to correspond. Such a change in these plates, with their necessarily small scale, would have brought confusion into the stress diagrams, from the necessity for entire redivision of the load line at every change in position of the rolling load.

The same remark applies to the construction of all the moment curves or polygons in a figure with one common side, and to the proportions of live and dead load here taken.

Further, some may object to considering the panel joint under the head of the moving load as fully loaded, while the one next in advance carries none of this moving load. This practice has its advantages in easier construction of diagrams, and many authors adopt it; the shearing stresses thus found are slightly in excess of the real maxima stresses, erring a trifle on the side of safety. A different distribution of the load can be used with a little more work.

One who makes himself familiar with the preceding graphical method will find that it possesses within itself many checks on the accuracy of the drawings, some of which have been alluded to. Those who prefer to find the stresses on the various parts of a truss by direct calculation will find a graphical method a valuable check on their computations, and a complete diagram, for every position of the moving load, shows readily to the eye, and more advantageously than the discussion and interpretation of mathematical formulæ would show to most persons, the increase and decrease of stresses in different members, the change of bending moment, the shifting of points of contra-flexure, and any other possible points of interest. Diagrams also afford a ready means for the comparison of trusses. This method can probably be extended to more problems of the same class, as it is en-

tirely general in its treatment. It has sometimes been thought desirable to make the moment over the piers, of a continuous girder, equal to the moment at the centre of the spans, and these diagrams may render that object attainable. The determination of the proportions of a stiffening girder for a suspension bridge can be made by this method.

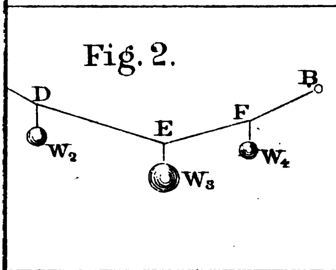


Fig. 2.

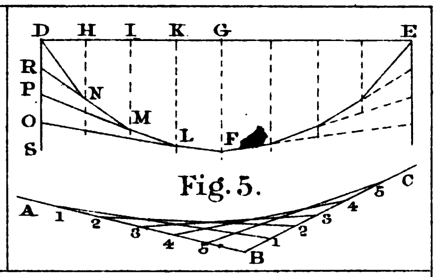
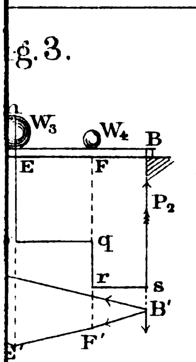


Fig. 5.



g. 3.

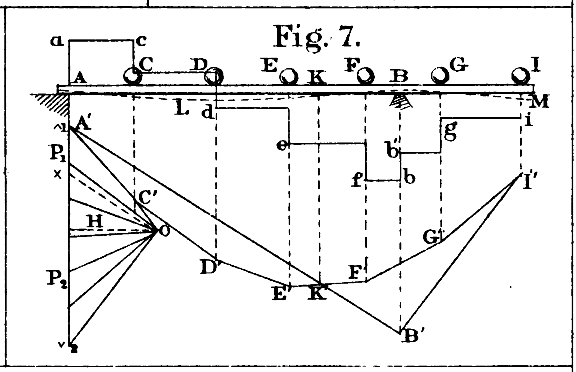
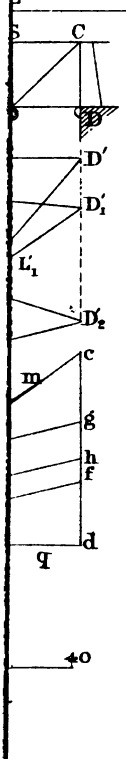


Fig. 7.



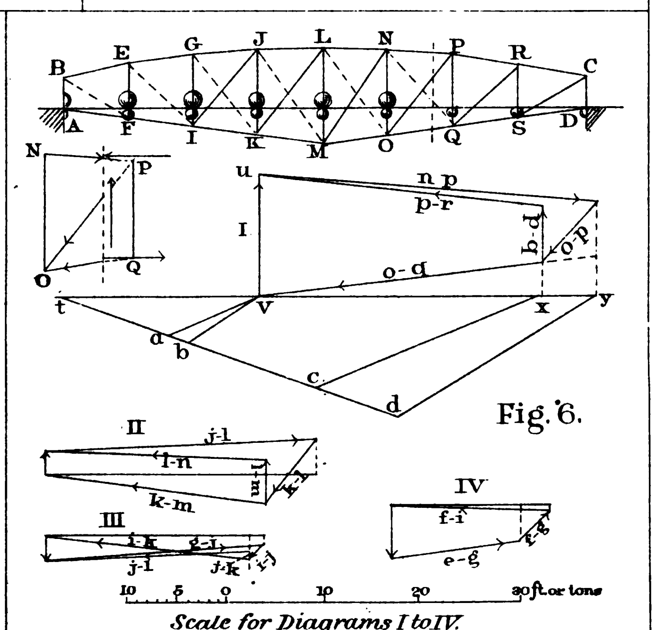


Fig. 6.

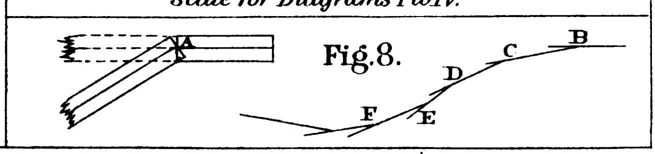


Fig. 8.

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